

Einstein-Podolsky-Rosen correlations in a hybrid system

Paweł Caban,^{*} Jakub Rembieliński,[†] P. Witas,[‡] and M. Włodarczyk[§]

Department of Theoretical Physics, University of Lodz

Pomorska 149/153, 90-236 Łódź, Poland

(ΩDated: December 14, 2010)

We calculate the relativistic correlation function for a hybrid system of a photon and a Dirac-particle. Such a system can be produced in decay of another spin-1/2 fermion. We show, that the relativistic correlation function, which depends on particle momenta, may have local extrema for fermion velocity of order $0.5c$. This influences the degree of violation of CHSH inequality.

PACS numbers: 03.65 Ta, 03.65 Ud

I. INTRODUCTION

In this paper we derive and study the Einstein-Podolsky-Rosen (EPR) correlations in a hybrid system of a massive spin-1/2 fermion and the photon in the relativistic regime. Investigation of the relativistic EPR correlation functions is important in view of certain tension between the spirit of special relativity and non-locality of quantum mechanics. This subject has been dealt with by numerous authors in recent years [1–29]. These studies have shown that the EPR correlations of relativistic massive particles differ from those of non-relativistic ones, namely the correlations depend on particle momenta and may show non-monotonic behavior. Moreover the latter are similar to the correlations involving only photons. Thus it is strongly desirable to measure the EPR correlations using relativistic massive particles [21].

To our knowledge, three experiments with the use of relativistic massive fermions have been performed so far. All the experiments measured proton-proton correlations. The first one, known as Laméhi-Rachti-Mittig experiment (LRM) [30], was performed in CEN-Saclay about thirty years ago. The LRM group tested Bell inequalities with the use of low-energy proton beams (13,5 MeV, that is $v \sim 0,17c$). The second experiment was conducted in Kernfysisch Versneller Instituut by S. Hamieh group [31]. In this experiment, spin correlations of proton pairs in 1S_0 state, emerging from $^{12}\text{C}(d, ^2\text{He})^{12}\text{B}$ nuclear charge-exchange reaction, were examined. Energy of protons was about 86 MeV ($v \sim 0.4c$). The third experiment was performed in RIKEN Accelerator Research Facility by H. Sakai group [32]. In the RIKEN experiment, proton pairs produced in $^1\text{H}(d, ^2\text{He})n$ reaction reached energy of 135 MeV ($v \sim 0.5c$). Results of these experiments are in agreement with the predictions of non-relativistic quantum mechanics. This is not surprising as the relativistic effects appear at much higher energies. For proton they are of the or-

der of 800 MeV ($v \sim 0.85c$) [24, 25]. However, it is an unprecedented difficulty to produce a singlet state and measure spin correlations of pairs of energetic protons. Thus it is fully justified to search for other processes that might allow a conclusive measurement. One of them are correlations in a hybrid system, that is in a system consisting of massive particles and photons.

In this paper we present a new concept to measure correlations in a simple hybrid system consisting of a massive spin-1/2 fermion and a photon. We show that for velocities $v \gtrsim 0.5c$ the fermion-photon correlation function depends non-monotonically on particle momenta. We also find that the violation of the Clauser-Horne-Shimony-Holt (CHSH) inequalities depends on particle momenta in non-monotonic way.

The paper is organized as follows. In Sec. II we construct the hybrid states. In Sec. III we define observables of polarization (for photon) and relativistic spin (for fermion). Then we calculate the correlation function (Sec. IV) and analyze the CHSH inequalities in some special cases (Sec. V). Conclusions are given in the Sec. VI.

II. CONSTRUCTION OF THE STATE OF THE HYBRID SYSTEM

We shall consider the EPR-type correlations in a bipartite system of the photon and a spin-1/2 fermion of mass m . Such a system can arise from a decay of another spin-1/2 fermion (of mass M).

Let q denote the four-momentum of the initial particle while k and p —the four-momenta of the final particles, i. e. photon and spin-1/2 fermion, respectively ($q = k + p$).

Let \mathcal{H}_f be the carrier space of the irreducible representation of the Poincaré group for fermion and \mathcal{H}_γ for photon. \mathcal{H}_f is spanned by the four-momentum operator eigenvectors, $|p, \sigma\rangle$, and \mathcal{H}_γ by $|k, \lambda\rangle$; σ denotes here the spin projection on z axis and λ —helicity for the photon. The above states do not transform in a manifestly covariant way [see (A1) and (A10) in Appendix A], but we can introduce states which fulfill this requirement.

In the present paper we use the notation from our previous papers [20, 21]. Summary of the notation is given in Appendix A. To keep the notation as simple as possible we denote the covariant states by kets with entries

^{*}Electronic address: P.Caban@merlin.phys.uni.lodz.pl

[†]Electronic address: J.Rembielinski@merlin.phys.uni.lodz.pl

[‡]Electronic address: pjwitas@gmail.com

[§]Electronic address: marta.wlodarczyk@gmail.com

ordered as follows: a Lorentz representation index (α – bispinor, μ – four-vector) followed by a four-momentum, i.e. $|\alpha, p\rangle$, $|(\mu, \nu), k\rangle$. Notice that the covariant vectors and the basis vectors of the representation space of the Poincaré group $|p, \sigma\rangle$, $|k, \lambda\rangle$, are distinguished by the order of entries; explicit relationship between both kinds of vectors is given below.

- For the spin-1/2 fermion

$$|\alpha, p\rangle = v_{\alpha\sigma}(p) |p, \sigma\rangle, \quad (1a)$$

$$U(\Lambda) |\alpha, p\rangle = D(\Lambda^{-1})_{\alpha\beta} |\beta, \Lambda p\rangle, \quad (1b)$$

where $D(\Lambda)$ is the bispinor $D^{(1/2,0)} \oplus D^{(0,1/2)}$ representation of the Lorentz group while Λ denotes matrix of an arbitrary proper orthochronous Lorentz transformation.

- For the photon

$$|(\mu, \nu), k\rangle = f^{\mu\nu}_{\lambda}(k) |k, \lambda\rangle, \quad (2a)$$

$$U(\Lambda) |(\mu, \nu), k\rangle = \Lambda^{-1\mu}_{\mu'} \Lambda^{-1\nu}_{\nu'} |(\mu', \nu'), \Lambda k\rangle, \quad (2b)$$

where

$$f^{\mu\nu}_{\lambda}(k) = k^{\mu} e^{\nu}_{\lambda}(k) - k^{\nu} e^{\mu}_{\lambda}(k). \quad (3)$$

Amplitudes $v_{\alpha\sigma}(p)$ and $e^{\mu}_{\lambda}(k)$ fulfill the Weinberg conditions [33, 34]; for their explicit forms see (A4), (A12).

The Hilbert space of the hybrid system is a tensor product space $\mathcal{H}_f \otimes \mathcal{H}_{\gamma}$ spanned by states $|p, \sigma\rangle \otimes |k, \lambda\rangle$. Because of transformation rules (1b) and (2b), the manifestly covariant basis states of the product space transform according to the $(D^{(1/2,0)} \oplus D^{(0,1/2)}) \otimes (D^{(1,0)} \oplus D^{(0,1)}) = D^{(1/2,0)} \oplus D^{(0,1/2)} \oplus D^{(1,1/2)} \oplus D^{(1/2,1)} \oplus D^{(3/2,0)} \oplus D^{(0,3/2)}$ representation of the Lorentz group. On the other hand, since the considered hybrid state $|\alpha; p, k\rangle$ arises from the decay of a spin-1/2 particle, it should transform according to the formula analogous to (1b)

$$U(\Lambda) |\alpha; p, k\rangle = D(\Lambda^{-1})_{\alpha\beta} |\beta; \Lambda p, \Lambda k\rangle. \quad (4)$$

In terms of the states (1a) and (2a), the basis vectors in the subspace of the hybrid states take the form:

$$|\alpha; p, k\rangle = [A_{\mu\nu}(p, k)]_{\alpha\beta} |\beta, p\rangle \otimes |(\mu, \nu), k\rangle. \quad (5)$$

These vectors should transform according to Eq. (4), which imposes constraints on $A_{\mu\nu}(p, k)$. By solving these constraints we find the following distinct bases:

$$|\alpha; p, k\rangle_1 = \frac{1}{4} [\gamma_{\mu}, \gamma_{\nu}]_{\alpha\beta} |\beta, p\rangle \otimes |(\mu, \nu), k\rangle, \quad (6a)$$

$$|\alpha; p, k\rangle_2 = (p_{\mu} \gamma_{\nu} - p_{\nu} \gamma_{\mu})_{\alpha\beta} |\beta, p\rangle \otimes |(\mu, \nu), k\rangle, \quad (6b)$$

and

$$|\alpha; p, k\rangle'_1 = \gamma_{\alpha\beta}^5 |\beta; p, k\rangle_1, \quad (7a)$$

$$|\alpha; p, k\rangle'_2 = \gamma_{\alpha\beta}^5 |\beta; p, k\rangle_2. \quad (7b)$$

The parity operator, $\hat{\mathcal{P}}$, acts on the vectors in the following way:

$$\hat{\mathcal{P}} |\alpha; p, k\rangle_{1/2} = \zeta^* \gamma_{\alpha\beta}^0 |\beta; p^{\pi}, k^{\pi}\rangle_{1/2}, \quad (8)$$

or

$$\hat{\mathcal{P}} |\alpha; p, k\rangle'_{1/2} = -\zeta^* \gamma_{\alpha\beta}^0 |\beta; p^{\pi}, k^{\pi}\rangle'_{1/2}, \quad (9)$$

where ζ is the inner parity of final state fermion. States (6) correspond to the case, when the inner parities of decaying and product spin-1/2 fermion are equal, and states (7) correspond to the case when decaying and product spin-1/2 fermions have opposite parities.

Hybrid states of a definite parity are combinations of the states (6) or (7).

Taking into account that the initial state of the decaying particle must fulfill the Dirac equation

$$(\hat{P}\gamma + M) |\Psi\rangle = 0, \quad (10)$$

where $\hat{P}\gamma = \hat{P}_{\mu} \gamma^{\mu}$, with \hat{P}_{μ} being the four-momentum operator and γ^{μ} the Dirac matrices (see Appendix A), we can determine the appropriate coefficients. As a result, we obtain:

$$\begin{aligned} |\Psi\rangle &= \bar{\Psi}_{\alpha} |\alpha; p, k\rangle \\ &\equiv \bar{\Psi}_{\alpha} [(m + M) |\alpha; p, k\rangle_1 + |\alpha; p, k\rangle_2], \end{aligned} \quad (11)$$

or

$$\begin{aligned} |\Psi\rangle' &= \bar{\Psi}_{\alpha} |\alpha; p, k\rangle' \\ &\equiv \bar{\Psi}_{\alpha} [(m - M) |\alpha; p, k\rangle'_1 + |\alpha; p, k\rangle'_2], \end{aligned} \quad (12)$$

where $\Psi = [\Psi_{\alpha}]$ is a bispinor and $\bar{\Psi} = \Psi^{\dagger} \gamma^0$.

Now, according to [17], we choose Ψ in such a way that the spin reduced density matrix $\Psi \bar{\Psi}$ describes the decaying particle with the polarization vector ξ

$$\Psi \bar{\Psi} = \rho = \frac{1}{8} \left(\mathbb{1} + \frac{q\gamma}{M} \right) \left(\mathbb{1} + 2\gamma^5 \frac{w(q)\gamma}{M} \right) \left(\mathbb{1} + \frac{q\gamma}{M} \right), \quad (13)$$

where the four-vector w

$$w^0 = \frac{\mathbf{q} \cdot \xi}{2}, \quad \mathbf{w} = \frac{1}{2} \left(M\xi + \frac{\mathbf{q}(\mathbf{q} \cdot \xi)}{M + q^0} \right) \quad (14)$$

is the mean value of the Pauli-Lubanski four-vector $\hat{W}^{\mu} = \frac{1}{2} \epsilon^{\nu\gamma\delta\mu} \hat{P}_{\nu} \hat{J}_{\gamma\delta}$ in the state ρ ; \hat{P}_{ν} and $\hat{J}_{\gamma\delta}$ denote the generators of the Poincaré group.

III. OBSERVABLES

In order to calculate the correlation function it is necessary to introduce the spin operator for a relativistic massive particles and the polarization operator for the photon.

Regarding the relativistic spin operator, several possibilities have been discussed in literature (see e.g. [1, 2, 5, 6, 8–10, 12, 13, 17, 20, 28, 35]). We restrict our considerations to the relativistic spin operator

$$\hat{\mathbf{S}} = \frac{1}{m} \left(\hat{\mathbf{W}} - \hat{W}^0 \frac{\hat{\mathbf{P}}}{\hat{P}^0 + m} \right), \quad (15)$$

which acts on one-particle states as follows:

$$\hat{\mathbf{S}} |p, \sigma\rangle = \frac{\boldsymbol{\sigma} \sigma' \sigma}{2} |p, \sigma'\rangle, \quad (16)$$

where σ_i are the Pauli matrices.

The appropriately normalized polarization observable is given by the obvious formula

$$\hat{\mathbf{S}}(\theta) = \frac{1}{2k^0 \delta^3(0)} (|\boldsymbol{\epsilon}_\theta, k\rangle \langle \boldsymbol{\epsilon}_\theta, k| - |\boldsymbol{\epsilon}_{\theta_\perp}, k\rangle \langle \boldsymbol{\epsilon}_{\theta_\perp}, k|), \quad (17)$$

where

$$|\boldsymbol{\epsilon}_\theta, k\rangle = \frac{1}{\sqrt{2}} (e^{i\theta} |k, +1\rangle + e^{-i\theta} |k, -1\rangle) \quad (18)$$

is the state of the linearly polarized photon with four-momentum k and $|\boldsymbol{\epsilon}_{\theta_\perp}, k\rangle = |\boldsymbol{\epsilon}_{\theta+\pi/2}, k\rangle$. The vector $|\boldsymbol{\epsilon}_\theta, k\rangle$ describes the photon polarized in the plane spanned by the vectors \mathbf{k} and $\boldsymbol{\epsilon}_\theta \perp \mathbf{k}$, where:

$$\boldsymbol{\epsilon}_\theta = \frac{1}{\sqrt{2}} \sum_\lambda \mathbf{e}_\lambda(k) e^{-i\lambda\theta}, \quad (19a)$$

$$\boldsymbol{\epsilon}_{\theta_\perp} = \frac{-i}{\sqrt{2}} \sum_\lambda \lambda \mathbf{e}_\lambda(k) e^{-i\lambda\theta}. \quad (19b)$$

The operator $\hat{\mathbf{S}}(\theta)$ acts on one-particle states as follows:

$$\hat{\mathbf{S}}(\theta) |k, \lambda\rangle = \sum_{\lambda'} \frac{1 - \lambda\lambda'}{2} e^{i(\lambda' - \lambda)\theta} |k, \lambda'\rangle. \quad (20)$$

IV. CORRELATION FUNCTION

Let us consider two distant observers, Alice and Bob, in the same inertial frame, sharing the state $|\Psi\rangle$. Let Bob measure polarization of the photon and Alice—spin projection of the fermion on some arbitrary direction \mathbf{a} , where $|\mathbf{a}| = 1$. As Alice assigns values ± 1 instead of $\pm 1/2$ to the outcomes of her measurement, her observable is $2\mathbf{a} \cdot \hat{\mathbf{S}}$. The correlation function takes the form

$$C_\Psi(\theta, \mathbf{a}, k, p) = \frac{\langle \Psi | 2\mathbf{a} \cdot \hat{\mathbf{S}} \otimes \hat{\mathbf{S}}(\theta) | \Psi \rangle}{\langle \Psi | \Psi \rangle}. \quad (21)$$

Inserting (11) into the above formula one gets:

$$C_\Psi(\theta, \boldsymbol{\xi}, \mathbf{a}, k, p) = \frac{m}{32M(kp)^2} \sum_\lambda e^{-2i\lambda\theta} \text{Tr} \left\{ [-(m+M)(k\gamma)(e_\lambda\gamma) + 2(kp)(e_\lambda\gamma) - 2(e_\lambda p)(k\gamma)] (\mathbb{1} + 2\gamma^5 \frac{w(q)\gamma}{M}) \right. \\ \left. \times [(m+M)(k\gamma)(e_\lambda\gamma) + 2(kp)(e_\lambda\gamma) - 2(e_\lambda p)(k\gamma)] v(p)(\mathbf{a} \cdot \boldsymbol{\sigma})^T \bar{v}(p) \right\}. \quad (22)$$

Now, taking into account the representation of the Dirac gamma matrices (A6) and Eq. (A4), we have

$$v(p)(\mathbf{a} \cdot \boldsymbol{\sigma})^T \bar{v}(p) = \frac{1}{2m} \left\{ - \left(m\mathbf{a} + \frac{\mathbf{a} \cdot \mathbf{p}}{m+p^0} \mathbf{p} \right) \gamma \gamma^5 \right. \\ \left. + (\mathbf{a} \cdot \mathbf{p}) \gamma^0 \gamma^5 - i[(\mathbf{a} \times \mathbf{p}) \boldsymbol{\gamma}] \gamma^0 + \left(p^0 \mathbf{a} - \frac{\mathbf{a} \cdot \mathbf{p}}{m+p^0} \mathbf{p} \right) \gamma \gamma^0 \gamma^5 \right\}. \quad (23)$$

Using the trace properties of the Dirac matrices we finally get

$$C(\theta, \boldsymbol{\xi}, \mathbf{a}, k, p) = \frac{(\boldsymbol{\alpha} \cdot \boldsymbol{\epsilon}_\theta)(\boldsymbol{\beta} \cdot \boldsymbol{\epsilon}_\theta) - (\boldsymbol{\alpha} \cdot \boldsymbol{\epsilon}_{\theta_\perp})(\boldsymbol{\beta} \cdot \boldsymbol{\epsilon}_{\theta_\perp})}{(kp)^2(M + k^0 + p^0)}, \quad (24)$$

where the polarization vectors are given by Eqs. (19) and we use the following notation

$$\boldsymbol{\alpha} := (kp)(M + p^0 + k^0) \boldsymbol{\xi} \\ + [(M + p^0)(\mathbf{k} \cdot \boldsymbol{\xi}) - k^0(\mathbf{p} \cdot \boldsymbol{\xi})] \mathbf{p}, \quad (25)$$

$$\boldsymbol{\beta} := (kp)\mathbf{a} + \left[(\mathbf{a} \cdot \mathbf{k}) - \frac{k^0(\mathbf{a} \cdot \mathbf{p})}{m+p^0} \right] \mathbf{p}, \quad (26)$$

(note that $kp = (M^2 - m^2)/2$). The formula (24) is valid also in the case of mixed states (when $|\xi| < 1$ instead of $|\xi| = 1$). The correlation function computed for the state $|\Psi\rangle'$ (12) differs from (24) by the overall sign.

Now, let us consider the correlation function in the center-of-mass frame (c.m. frame), i. e. $\mathbf{p} = -\mathbf{k}$. The components k^0 and p^0 read

$$k^0 = \frac{M^2 - m^2}{2M}, \quad p^0 = \frac{M^2 + m^2}{2M}, \quad (27)$$

and the components of the four-vector w (14) take the form:

$$w^0 = 0, \quad \mathbf{w} = \frac{M\xi}{2}. \quad (28)$$

In such a frame, the correlation function reduces to

$$C_{\text{c.m.}}(\theta, \xi, \mathbf{a}) = (\mathbf{a} \cdot \epsilon_\theta)(\xi \cdot \epsilon_\theta) - (\mathbf{a} \cdot \epsilon_{\theta_\perp})(\xi \cdot \epsilon_{\theta_\perp}), \quad (29)$$

where the polarization vectors are defined by Eqs. (19). As we can see the function (29) does not depend on the value of particle momenta.

In further considerations we have adopted the following parametrization of the vectors \mathbf{a} and \mathbf{p} :

$$\mathbf{a} = \begin{pmatrix} \cos \varsigma \\ \sin \varsigma \sin \varphi \\ \sin \varsigma \cos \varphi \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} |\mathbf{p}| \cos \psi \\ |\mathbf{p}| \sin \psi \\ 0 \end{pmatrix}. \quad (30)$$

Without loss of generality we have assumed that the photon propagates along x axis, i. e. $\mathbf{k} = (|\mathbf{k}|, 0, 0)$. In that case the vectors (19) take the form

$$\epsilon_\theta = \begin{pmatrix} 0 \\ \sin \theta \\ \cos \theta \end{pmatrix}, \quad \epsilon_{\theta_\perp} = \begin{pmatrix} 0 \\ \cos \theta \\ -\sin \theta \end{pmatrix}. \quad (31)$$

After setting $\xi = (0, 0, 1) \equiv \xi_0$ (see Fig. 1), the correlation function (24) takes the form

$$C(\theta, \xi_0, \mathbf{a}, k, p) = \sin \varsigma \cos(\varphi - 2\theta) + \frac{\sqrt{x}}{\sqrt{x+1} - \sqrt{x} \cos \psi} \left[\cos \varsigma - \frac{\sqrt{x}}{\sqrt{x+1} + 1} (\cos \varsigma \cos \psi + \sin \varsigma \sin \psi \sin \varphi) \right] \sin \psi \sin 2\theta, \quad (32)$$

where $x = \left(\frac{|\mathbf{p}|}{m}\right)^2$.

We show the x -dependence of the function (32) for two sets of parameters: $\varsigma = 2\pi/3$, $\varphi = 3\pi/2$, $\psi = \pi/3$, $\theta = \pi/4$ (Fig 2) and $\varsigma = \varphi = \pi/4$, $\psi = \theta = \pi/3$ (Fig 3). The function (32) has extremum in both cases, a minimum and a maximum, respectively: $C = -0.87$ for $x = 1/3$ and $C = 0.5$ for $x = 1.36$. Such a property of the correlation function in a bipartite systems of a vector bosons and spin-1/2 fermions has already been reported by us [22–24, 26]. Moreover, it also influences the degree of violation of CHSH inequalities. One

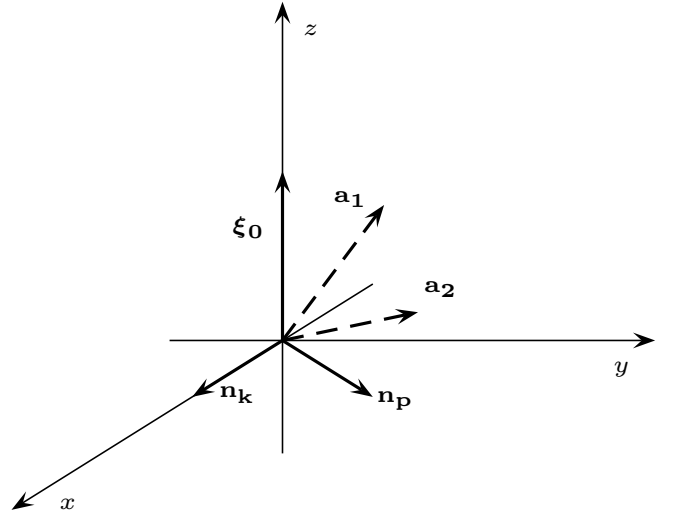


FIG. 1: Configuration in which the correlation function (32) was calculated. Vector ξ [see Eq. (14)] lies on the z axis, the photon propagates along the x axis, the momentum of the spin-1/2 fermion (\mathbf{n}_p) lies on the xy plane and the directions on which Bob measures spin projections of the fermion are denoted by \mathbf{a}_1 and \mathbf{a}_2 . In the c.m. frame $\mathbf{n}_k = -\mathbf{n}_p$.

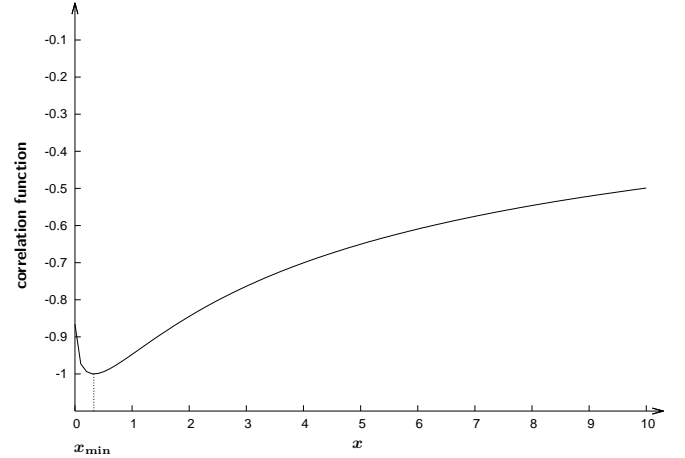


FIG. 2: Dependence of the correlation function $C(\theta, \xi_0, \mathbf{a}, k, p)$ on $x = \left(\frac{|\mathbf{p}|}{m}\right)^2$ for $\varsigma = 2\pi/3$, $\varphi = 3\pi/2$, $\psi = \pi/3$ and $\theta = \pi/4$. The value of the minimum equals -0.87 ($x_{\min} = 1/3$).

should notice, that the velocities that enable observation of relativistic effects are for a bipartite proton system of about $0.85c$ [25]. For hybrid system, extrema appear for $x = 1/3$, i. e. for $v = 0.5c$, which is the border that was reached in RIKEN experiment [32].

The ultra-relativistic limit of function (32) reads

$$C_{\text{ultrarel}}(\theta, \xi_0, \mathbf{a}) = \sin \varsigma \cos \varphi \cos 2\theta + \sin 2\theta (\cos \varsigma \sin \psi - \sin \varsigma \cos \psi \sin \varphi). \quad (33)$$

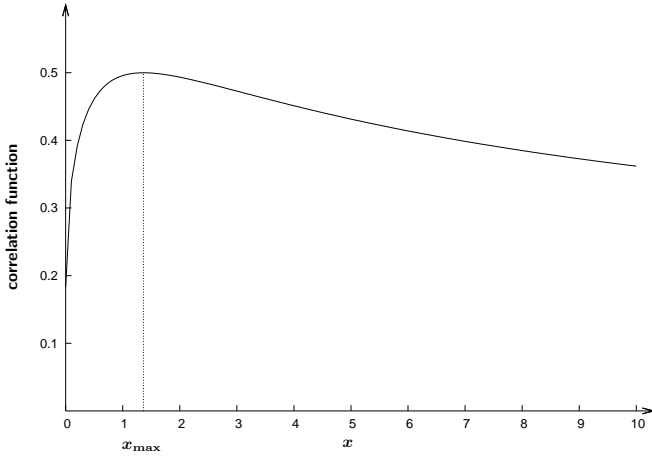


FIG. 3: Dependence of the correlation function $C(\theta, \xi_0, \mathbf{a}, k, p)$ on $x = \left(\frac{|\mathbf{p}|}{m}\right)^2$ for $\varsigma = \varphi = \pi/4$ and $\psi = \theta = \pi/3$. The value at the maximum equals $1/2$ ($x_{\max} = 1.36$).

The non-relativistic limit has the form

$$C_{\text{nonrel}}(\theta, \xi_0, \mathbf{a}) = \sin \varsigma \cos(\varphi - 2\theta), \quad (34)$$

which is exactly equal to the form of correlation function (29), $C_{\text{c.m.}}(\theta, \xi, \mathbf{a}) = C_{\text{nonrel}}(\theta, \xi_0, \mathbf{a})$, in configuration we chose. Furthermore the function (34) has the same form as (B13) up to normalization.

V. THE CHSH INEQUALITY

In this section we consider the CHSH inequality and show, that it can be violated in arbitrary reference frame.

We search for the configuration, that maximally violates the CHSH inequality which reads:

$$|C(\theta_1, \xi, \mathbf{a}_1, \mathbf{k}, \mathbf{p}) + C(\theta_1, \xi, \mathbf{a}_2, \mathbf{k}, \mathbf{p}) + C(\theta_2, \xi, \mathbf{a}_2, \mathbf{k}, \mathbf{p}) - C(\theta_2, \xi, \mathbf{a}_1, \mathbf{k}, \mathbf{p})| \leq 2. \quad (35)$$

Just like in previous section, we assume that the photon propagates along the x axis and ξ is orientated along the z axis (Fig. 1).

The correlation function (32) violates the CHSH inequalities, moreover, the degree of violation is related to the existence of the extrema. This is shown in Fig. 4 where the left hand side of (35) is plotted versus the variable x . We see, that the inequality is violated in non-relativistic case, then the degree of its violation increases to reach 2.60 at $x_{\max} = 0.71$. Then it monotonically decreases, and for $x > 6.38$ the inequality is satisfied. In Fig. 5 the inequality is satisfied for the non-relativistic case and violated for $x \in (0.21, 6.54)$. It has maximum equal to 2.28 at $x_{\max} = 1.77$. In Fig. 6, the left hand side of the CHSH inequality is always greater than 2. In non-relativistic case the inequality is maximally violated and then the degree of violation monotonically decreases.

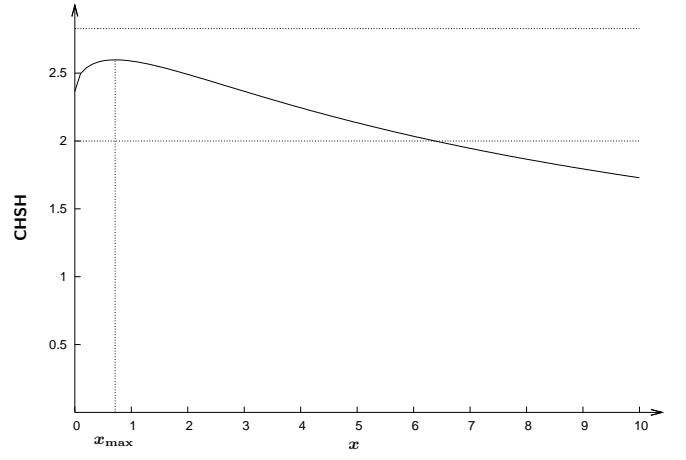


FIG. 4: Dependence of the left hand side of CHSH inequality (35) on x for $\psi = \pi/6$, $\varsigma_1 = 2\pi/3$, $\varphi_1 = 3\pi/2$, $\theta_1 = 2\pi/3$, $\varsigma_2 = \pi/3$, $\varphi_2 = \pi$ and $\theta_2 = \pi/3$. It has maximum equal to 2.60 at $x_{\max} = 0.71$. The inequality is violated for $x < 6.38$.

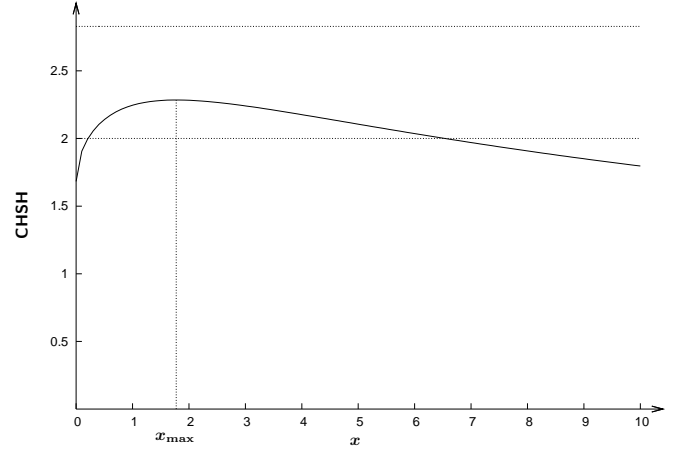


FIG. 5: Dependence of the left hand side of CHSH inequality (35) on x for $\psi = \varsigma_1 = \pi/6$, $\varphi_1 = \pi/2$, $\theta_1 = 3\pi/4$, $\varsigma_2 = \varphi_2 = \pi/3$ and $\theta_2 = \pi/2$. It has maximum equal to 2.28 at $x_{\max} = 1.77$. The inequality is violated for $x \in (0.21, 6.54)$.

In c.m. frame, the CHSH inequality takes the form:

$$2|\sin \varsigma_1 \sin(\varphi_1 - \theta_1 - \theta_2) \sin(\theta_1 - \theta_2) + \sin \varsigma_2 \cos(\varphi_2 - \theta_1 - \theta_2) \cos(\theta_1 - \theta_2)| \leq 2. \quad (36)$$

It follows from the above formula that in the relativistic case it is possible to get the maximal violation of the CHSH inequality ($2\sqrt{2}$), for example when we set $\varsigma_1 = \varsigma_2 = \pi/2$ and $\theta_1 - \theta_2 = \pi/4$. Thus, when $\varphi_1 - \theta_1 - \theta_2 = \pi/2$ and $\varphi_2 - \theta_1 - \theta_2 = 0$ (e.g. for $\varphi_1 = 3\pi/4$, $\varphi_2 = \pi/4$, $\theta_1 = \pi/4$ and $\theta_2 = 0$) the left hand side of (36) is equal to $2\sqrt{2}$.

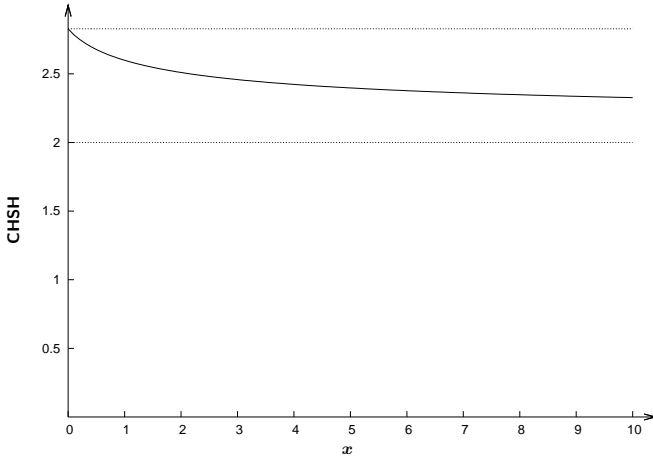


FIG. 6: Dependence of the left hand side of CHSH inequality (35) on x for $\psi = 2\pi/3$, $\varsigma_1 = \pi/2$, $\varphi_1 = 3\pi/4$, $\theta_1 = \pi/4$, $\varsigma_2 = \pi/2$, $\varphi_2 = \pi/4$ and $\theta_2 = 0$. The inequality is violated in whole domain, the maximal violation occurs in non-relativistic case.

VI. CONCLUSIONS

We have constructed quantum state of a hybrid system (a massive spin-1/2 fermion and the photon) arising from the decay of another spin-1/2 fermion. The constructed state is characterized by the polarization (Bloch) vector ξ [see Eq. (13)]. We have calculated the correlation function in the EPR-type experiment with hybrid system assuming that Alice measures the spin projection of the fermion and Bob the polarization of the photon. Next we have analyzed the correlation function in some configurations and found that it can be a non-monotonic function of the momentum of the fermion. Similar behavior of the correlation function has been reported in the case of bipartite fermion (or boson) system. However, in a hybrid system extrema occur for lower velocity of the fermion ($0.5c$) than in the two-fermion system ($0.85c$). We have analyzed the CHSH inequality, too. We have found the configuration in which the inequality is violated maximally. Moreover, we have shown that the degree of violation of the CHSH inequality can be a non-monotonic function of the fermion momentum. We have also compared the results with non-relativistic case.

Let us note that a system fermion+photon can be produced also in the Compton scattering. It seems that states prepared in this way are easier to handle experimentally. Theoretical analysis of the correlations in this case is more complicated and will be given in the subsequent paper.

Acknowledgments

The authors are grateful to Jacek Ciborowski for interesting discussion. This work has been supported by the University of Lodz and by the Polish Ministry of Science

and Higher Education under the contract N N202 103738. P. W. was supported by the European Union under the European Social Fund: Human - best investment.

Appendix A: Poincaré representations of spin-1/2 fermion and a photon

For readers convenience and to establish the notation, we recall some basic facts and formulas necessary to follow the paper.

1. Spin-1/2 fermion

The notation and formalism we use in the case of spin-1/2 particles is explained wider in our previous paper [20]. Here we briefly recall the most important points. For spin-1/2 fermion, the carrier space of irreducible representation of the Poincaré group is spanned by the four-momentum operator eigenvectors $|p, \sigma\rangle$, where $p^2 = m^2$, with m denoting the mass of the particle, and $\sigma = \pm 1/2$. Their transformation rule reads

$$U(\Lambda) |q, \sigma\rangle = \mathcal{D}_{\sigma'\sigma}(R(\Lambda, q)) |\Lambda q, \sigma'\rangle, \quad (\text{A1})$$

where \mathcal{D} is the spin 1/2 fundamental representation of $SU(2)$, and the Wigner rotation $R(\Lambda, q) \in SO(3)$ is defined as $R(\Lambda, q) = L_{\Lambda q}^{-1} \Lambda L_q$. We use Lorentz-covariant normalization

$$\langle p, \sigma | p', \sigma' \rangle = 2p^0 \delta^3(\mathbf{p} - \mathbf{p}') \delta_{\sigma\sigma'}. \quad (\text{A2})$$

Consistency of Eqs. (A1, 1a, 1b) leads to Weinberg condition

$$D(\Lambda) v(p) \mathcal{D}^T(R(\Lambda, p)) = v(\Lambda p). \quad (\text{A3})$$

For D being a bispinor representation $D^{(1/2,0)} \oplus D^{(0,1/2)}$ of the Lorentz group, we can find (see e.g. [17])

$$v(p) = \frac{1}{2\sqrt{1 + \frac{p^0}{m}}} \begin{pmatrix} (\mathbb{1} + \frac{1}{m} p^\mu \sigma_\mu) \sigma_2 \\ (\mathbb{1} + \frac{1}{m} (p^\pi)^\mu \sigma_\mu) \sigma_2 \end{pmatrix}, \quad (\text{A4})$$

where $\sigma_0 \equiv \mathbb{1}$, σ_i are standard Pauli matrices and $p^\pi = (p^0, -\mathbf{p})$. The amplitudes (A4) fulfill

$$v(p) \bar{v}(p) = \frac{1}{2m} (m + p\gamma), \quad (\text{A5a})$$

$$\bar{v}(p) v(p) = \mathbb{1}, \quad (\text{A5b})$$

where $p\gamma = p_\mu \gamma^\mu$ and $\bar{v}(p) = v^\dagger(p) \gamma^0$ stands for Dirac conjugation. We use the following explicit representation of gamma matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 0 & -\sigma \\ \sigma & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad (\text{A6})$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. It holds

$$\langle \delta, p | \gamma_{\delta\alpha}^0 | \beta, p' \rangle = \frac{p^0}{m} \delta^3(\mathbf{p} - \mathbf{p}') [m + p\gamma]_{\beta\alpha}. \quad (\text{A7})$$

The action of the space inversion operator $\hat{\mathcal{P}}$ on the basis states is given by

$$\hat{\mathcal{P}} |p, \sigma\rangle = \zeta^* |p^\pi, \sigma\rangle, \quad (\text{A8})$$

consequently for covariant states defined by (1a)

$$\hat{\mathcal{P}} |\alpha, p\rangle = \zeta^* \gamma_{\alpha\beta}^0 |\beta, p^\pi\rangle, \quad (\text{A9})$$

with ζ denoting the inner parity of the particle.

2. Photon

The notation and formalism we use in the case of photons is explained wider in our previous paper [21]. Here we briefly recall the most important points. The carrier space of the irreducible photon representation of the Poincaré group is spanned by four-momentum eigenstates $|k, \lambda\rangle$ with $\lambda = \pm 1$ denoting helicity and $k^2 = 0$. Their transformation rule reads

$$U(\Lambda) |k, \lambda\rangle = e^{i\lambda\psi(\Lambda, k)} |\Lambda k, \lambda\rangle, \quad (\text{A10})$$

where $e^{i\lambda\psi(\Lambda, k)} = U(R(\Lambda, k))$.

They are normalized as follows:

$$\langle k, \lambda | k', \lambda' \rangle = 2p^0 \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\lambda\lambda'}. \quad (\text{A11})$$

The vectors $|k, \lambda\rangle$ can be generated from standard vector $|\tilde{k}, \lambda\rangle$, with $\tilde{k} = (1, 0, 0, 1)$. We have $|k, \lambda\rangle = U(L_k) |\tilde{k}, \lambda\rangle$, where $L_k = R_{\mathbf{n}_k} B(k^0)$. $B(k^0)$ is a pure Lorentz boost taking vector \tilde{k} into $k^0 \tilde{k}$ and $R_{\mathbf{n}_k}$ is rotation transforming vector \tilde{k} into $(1, \mathbf{n}_k)$, where $\mathbf{n}_k = \mathbf{k}/|\mathbf{k}|$.

The explicit form of amplitudes $e_\lambda(k)$, which define the covariant state (2) reads

$$[e_{\mu\lambda}(k)] = \frac{1}{\sqrt{2}} R_{\mathbf{n}_k} \begin{pmatrix} 0 \\ -1 \\ i\lambda \\ 0 \end{pmatrix}. \quad (\text{A12})$$

The general form of $R_{\mathbf{n}_k}$ is [23]

$$R_{\mathbf{n}_k} = \left(\begin{array}{c|c} 1 & \mathbf{0}^T \\ \hline \mathbf{0} & \mathbf{a}_k | \mathbf{n}_k \times \mathbf{a}_k | \mathbf{n}_k \end{array} \right), \quad (\text{A13})$$

where $\mathbf{a}_k \perp \mathbf{n}_k$, $|\mathbf{a}_k| = 1$.

The choice of \mathbf{a}_k is the matter of convention. Without loss of generality we can assume that photon propagates along x axis and then choose $\mathbf{a}_k = (0, 0, 1)$, which is the standard choice we use in this paper.

The action of the space inversion operator $\hat{\mathcal{P}}$ on the basis states is:

$$\hat{\mathcal{P}} |k, \lambda\rangle = \chi_\lambda(k) |k^\pi, -\lambda\rangle, \quad (\text{A14})$$

and consequently for covariant states defined by (2a)

$$\hat{\mathcal{P}} |(\mu, \nu), k\rangle = \eta^{\mu\mu} \eta^{\nu\nu} |(\mu, \nu), k^\pi\rangle, \quad (\text{A15})$$

with

$$\mathbf{e}_\lambda(k) = -\chi_{-\lambda}(k^\pi) \mathbf{e}_{-\lambda}(k^\pi) \quad (\text{A16})$$

and

$$\chi_\lambda(k) \chi_{-\lambda}(k^\pi) = 1. \quad (\text{A17})$$

In the convention used above $\chi_\lambda(k) = -1$.

Coefficients $f_\lambda^{\mu\nu}$ defined by Eq. (3) fulfill

$$f_\lambda^{*\mu\nu} f_\lambda^{\mu'\nu'} = \eta^{\mu\nu'} k^\nu k^{\mu'} + \eta^{\nu\mu'} k^\mu k^{\nu'} - \eta^{\mu\mu'} k^\nu k^{\nu'} - \eta^{\nu\nu'} k^\mu k^{\mu'}, \quad (\text{A18})$$

where $\eta = \text{diag}(1, -1, -1, -1)$.

Appendix B: Non-relativistic case

As a non-relativistic analog of a hybrid system we take the system consisting of a spin-1/2 fermion and a spin-1 massive boson. The total spin of the system is equal to 1/2. Using the Clebsh-Gordan coefficients we can write down two linearly independent states of the system with total spin equal to 1/2

$$|\uparrow\rangle = \sqrt{\frac{2}{3}} |-1/2\rangle_f \otimes |1\rangle_b - \sqrt{\frac{1}{3}} |1/2\rangle_f \otimes |0\rangle_b, \quad (\text{B1a})$$

$$|\downarrow\rangle = \sqrt{\frac{1}{3}} |-1/2\rangle_f \otimes |0\rangle_b - \sqrt{\frac{2}{3}} |1/2\rangle_f \otimes |-1\rangle_b, \quad (\text{B1b})$$

where $|\sigma\rangle_{f/b}$ stands for the state of a fermion/boson with spin projection on z axis equal to σ .

Therefore, the general state of the system with total spin equal to 1/2 has the following form

$$|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle. \quad (\text{B2})$$

The density matrix corresponding to the above state reads

$$|\psi\rangle \langle\psi| = \frac{1}{2} [(|\uparrow\rangle \langle\uparrow| + |\downarrow\rangle \langle\downarrow|) + \xi_1 (|\downarrow\rangle \langle\uparrow| + |\uparrow\rangle \langle\downarrow|) + i\xi_2 (|\downarrow\rangle \langle\uparrow| - |\uparrow\rangle \langle\downarrow|) + \xi_3 (|\uparrow\rangle \langle\uparrow| - |\downarrow\rangle \langle\downarrow|)], \quad (\text{B3})$$

where the components of the Bloch vector $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ are connected with the coefficients α, β from Eq. (B2) by

$$|\alpha|^2 = (1 + \xi_3)/2, \quad (\text{B4a})$$

$$|\beta|^2 = (1 - \xi_3)/2, \quad (\text{B4b})$$

$$\alpha\beta^* = (\xi_1 - i\xi_2)/2. \quad (\text{B4c})$$

Note, that in the subspace spanned by the vectors $|\uparrow\rangle$ and $|\downarrow\rangle$, matrix elements of the operators $(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$, $(|\downarrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\downarrow|)$, $i(|\downarrow\rangle\langle\uparrow| - |\uparrow\rangle\langle\downarrow|)$, $(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$ are the same as the matrix elements of the Pauli matrices σ_0 , σ_1 , σ_2 , and σ_3 .

We want to calculate the non-relativistic correlation function

$$\frac{\langle\psi|2\mathbf{a}\cdot\hat{\mathbf{S}}\otimes\hat{S}_\theta|\psi\rangle}{\langle\psi|\psi\rangle}, \quad (\text{B5})$$

where $\mathbf{a}\cdot\hat{\mathbf{S}}$ is an observable measuring spin projection of a spin-1/2 particle on the direction \mathbf{a} , i. e.

$$\mathbf{a}\cdot\hat{\mathbf{S}}|\sigma\rangle_f = \frac{1}{2}\mathbf{a}\cdot\boldsymbol{\sigma}_{\lambda\sigma}|\lambda\rangle_f. \quad (\text{B6})$$

\hat{S}_θ is the polarization observable. It is convenient to define the polarization observable in terms of helicity basis $|\mathbf{n}_\mathbf{k}, \lambda\rangle$, with $\mathbf{n}_\mathbf{k} = \mathbf{k}/|\mathbf{k}|$ denoting the direction of spin-1 particle momentum and λ standing for its helicity. The helicity basis can be expressed by means of spin basis as follows:

$$|\mathbf{n}_\mathbf{k}, \lambda\rangle = \mathcal{D}_{\sigma\lambda}^{(1)}(R_{\mathbf{n}_\mathbf{k}})|\sigma\rangle_b, \quad (\text{B7})$$

where $R_{\mathbf{n}_\mathbf{k}}$ is given by Eq. (A13) and $\mathcal{D}^{(1)}$ denotes standard spin-1 representation of the rotation group (see e.g. [22]). In the above basis, the polarization observable is defined in analogy to Eq. (17) by

$$\hat{S}_\theta = |\epsilon_\theta, \mathbf{n}_\mathbf{k}\rangle\langle\epsilon_\theta, \mathbf{n}_\mathbf{k}| - |\epsilon_{\theta_\perp}, \mathbf{n}_\mathbf{k}\rangle\langle\epsilon_{\theta_\perp}, \mathbf{n}_\mathbf{k}|, \quad (\text{B8})$$

where

$$|\epsilon_\theta, \mathbf{n}_\mathbf{k}\rangle = \frac{1}{\sqrt{2}}(e^{i\theta}|\mathbf{n}_\mathbf{k}, +1\rangle + e^{-i\theta}|\mathbf{n}_\mathbf{k}, -1\rangle) \quad (\text{B9})$$

and $\theta_\perp = \theta + \pi/2$.

Let us now consider a configuration, when the three-momentum of the boson is along the x axis $[\mathbf{n}_\mathbf{k} = (1, 0, 0)]$, and $\mathbf{a}_\mathbf{k} = (0, 0, 1)$, where $\mathbf{a}_\mathbf{k}$ is the vector defining rotation (A13). In such a configuration

$$|\mathbf{n}_\mathbf{k}, 1\rangle = -\frac{1}{2}(|1\rangle_b + \sqrt{2}|0\rangle_b + |-1\rangle_b), \quad (\text{B10a})$$

$$|\mathbf{n}_\mathbf{k}, 0\rangle = \frac{1}{\sqrt{2}}(|1\rangle_b - |-1\rangle_b), \quad (\text{B10b})$$

$$|\mathbf{n}_\mathbf{k}, -1\rangle = \frac{1}{2}(|1\rangle_b - \sqrt{2}|0\rangle_b + |-1\rangle_b). \quad (\text{B10c})$$

Therefore

$$|\epsilon_\theta, \mathbf{n}_\mathbf{k}\rangle = -\frac{i\sin\theta}{\sqrt{2}}(|1\rangle_b + |-1\rangle_b) - \cos\theta|0\rangle_b. \quad (\text{B11})$$

In this case the correlation function in the state (B2) reads

$$C(\theta, \boldsymbol{\xi}, \mathbf{a}) = \frac{2}{3}[(a_3\xi_3 - a_2\xi_2)\cos 2\theta + (a_2\xi_3 + a_3\xi_2)\sin 2\theta]. \quad (\text{B12})$$

When we use parametrization (30) it reduces to

$$C(\theta, \boldsymbol{\xi}_0, \mathbf{a}) = \frac{2}{3}\sin\varsigma\cos(\varphi - 2\theta), \quad (\text{B13})$$

Note that the factor 2/3 in formula (B12) appears because the probabilities of that Bobs measurement outcome equals 0, enters the correlation function with measure 0. If we normalized all probabilities entering the correlation function to 1, the factor would be 1, just as in the relativistic case, where the probability of Bob getting the outcome 0 vanishes.

-
- [1] M. Czachor, Phys. Rev. A **55**, 72 (1997).
 - [2] J. Rembieliński and K. A. Smoliński, Phys. Rev. A **66**, 052114 (2002).
 - [3] R. M. Gingrich and C. Adami, Phys. Rev. Lett. **89**, 270402 (2002).
 - [4] R. M. Gingrich, A. J. Bergou, and C. Adami, Phys. Rev. A **68**, 042102 (2003).
 - [5] H. Terashima and M. Ueda, Int. J. Quantum Information **1**, 93 (2003).
 - [6] H. Terashima and M. Ueda, Quantum Information and Computation **3**, 224 (2003).
 - [7] D. Ahn, H. J. Lee, Y. H. Moon, and S. W. Hwang, Phys. Rev. A **67**, 012103 (2003).
 - [8] D. R. Terno, Phys. Rev. A **67**, 014102 (2003).
 - [9] M. Czachor and M. Wilczewski, Phys. Rev. A **68**, 010302(R) (2003).
 - [10] H. Li and J. Du, Phys. Rev. A **68**, 022108 (2003).
 - [11] P. Caban and J. Rembieliński, Phys. Rev. A **68**, 042107 (2003).
 - [12] D. Lee and E. Chang-Young, New Journal of Physics **6**, 67 (2004).
 - [13] C. Soo and C. C. Y. Lin, Int. J. Quantum Information **2**, 183 (2004).
 - [14] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93 (2004).
 - [15] H. You, A. M. Wang, X. Young, W. Niu, X. Ma, and F. Xu, Phys. Lett. A **333**, 389 (2004).
 - [16] M. Czachor, Phys. Rev. Lett. **94**, 078901 (2005).
 - [17] P. Caban and J. Rembieliński, Phys. Rev. A **72**, 012103 (2005).
 - [18] A. Peres, P. F. Scudo, and D. R. Terno, Phys. Rev. Lett. **94**, 078902 (2005).
 - [19] W. T. Kim and E. J. Son, Phys. Rev. A **71**, 014102 (2005).
 - [20] P. Caban and J. Rembieliński, Phys. Rev. A **74**, 042103 (2006).
 - [21] P. Caban, Phys. Rev. A **76**, 052102 (2007).

- [22] P. Caban, Phys. Rev. A **77**, 062101 (2008).
- [23] P. Caban, J. Rembieliński, and M. Włodarczyk, Phys. Rev. A **77**, 012103 (2008).
- [24] P. Caban, J. Rembieliński, and M. Włodarczyk, Phys. Rev. A **79**, 014102 (2009).
- [25] J. Rembieliński and K. A. Smoliński, EPL **88**, 10005 (2009).
- [26] P. Caban, A. Dzięgielewska, A. Karmazyn, and M. Okrasa, Phys. Rev. A **81**, 032112 (2010).
- [27] N. Friis, R. A. Bertlmann, M. Huber, and B. Hiesmayr, Phys. Rev. A **81**, 042114 (2010).
- [28] M. Czachor, Quantum Information Processing **9**, 171 (2010).
- [29] P. Caban, J. Rembieliński, and M. Włodarczyk, arXiv:1011.5880.
- [30] M. Lamehi-Rachti and W. Mittig, Phys. Rev. D **14**, 2543 (1976).
- [31] S. Hamieh and et al., J. Phys. G: Nucl. Part. Phys. **30**, 481 (2004).
- [32] H. Sakai, Phys. Rev. Lett. **97**, 150405 (2006).
- [33] S. Weinberg, in *Lectures on Particles and Field Theory*, edited by S. Deser and K. W. Ford (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964), vol. II of *Lectures delivered at Brandeis Summer Institute in Theoretical Physics*, p. 405.
- [34] S. Weinberg, *The Quantum Theory of Fields*, vol. I (Cambridge University Press, Cambridge, 1996).
- [35] N. N. Bogolubov, A. A. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory* (Benjamin, Reading, MA, 1975).